AMC 10 / 12 COMPETITION MATH MASTER CHECKLIST

Complete Concept & Theorem Inventory — AoPS Volume 1 & Volume 2

Scope: Geometry, Combinatorics, Algebra, Number Theory

Total Topics: 65 Core Modules & Theorems

Target: Contest Perfection & Mastery

1. GEOMETRY

CORE CONCEPTS & FUNDAMENTALS

- [ ] **Angle Chasing:** Parallel line angles, cyclic quadrilaterals, exterior angle theorem, and isosceles setups.
- [ ] **Triangle Area & Length Formulas:** $A = \frac{1}{2}bh$, $A = \frac{1}{2}ab \sin C$, inradius $A = rs$, circumradius $A = abc / 4R$.
- [ ] **Special Triangles & Triples:** $30°$-$60°$-$90°$, $45°$-$45°$-$90°$; Triples: (3-4-5), (5-12-13), (8-15-17), (7-24-25), (20-21-29).
- [ ] **Similar Triangles:** Establishing ratios via AA, SAS, SSS; area ratio rule k^2 for length ratio k.
- [ ] **Quadrilaterals:** Parallelograms, trapezoids, rhombuses, kites, cyclic & tangential quadrilaterals.
- [ ] **Circles:** Tangents, secants, arc lengths, sector areas, circumscribed/inscribed circles, radical axis.
- [ ] **Polygons:** Interior angle sum $(n-2)180°$, exterior sum $360°$, diagonals count $n(n-3)/2$.
- [ ] **3D Geometry:** Prisms, pyramids, cones, spheres, space diagonals, and 2D cross-sections.
- [ ] **Complex Shapes & Shaded Regions:** Area and length calculations of composite, overlapping figure systems.
- [ ] **Coordinate Geometry:** Slopes, perpendicular lines ($m_1 \cdot m_2 = -1$), circles, distance & midpoint formulas.

GEOMETRY THEOREMS & FORMULAS

- [ ] **Pythagorean Theorem:** $a^2 + b^2 = c^2$ in 2D plane and 3D space.
- [ ] **Angle Bisector Theorem:** Internal side ratio equality $BD / DC = AB / AC$.
- [ ] **Heron's Formula:** $A = \sqrt{s(s-a)(s-b)(s-c)}$ where semiperimeter $s = (a+b+c)/2$.
- [ ] **Shoelace Formula:** Finding area of any polygon using ordered coordinate vertices (x_i, y_i).
- [ ] **Ceva's Theorem:** Concurrency condition for three cevians: $(AF/FB)\cdot(BD/DC)\cdot(CE/EA) = 1$.
- [ ] **Stewart's Theorem:** Cevian length determination: $man + dad = b^2n + c^2m$.
- [ ] **Power of a Point Theorem:** Chord-Chord ($AE\cdot EB = CE\cdot ED$), Tangent-Secant ($PT^2 = PA\cdot PB$), Secant-Secant.
- [ ] **Inscribed Angle Theorem:** Inscribed measure is exactly half of the central angle/intercepted arc.
- [ ] **Ptolemy's Theorem:** Diagonals and side relation in cyclic quadrilaterals: $ac + bd = ef$.
- [ ] **Pick's Theorem:** Lattice polygon area: $A = I + B/2 - 1$ (I = interior points, B = boundary points).

2. COMBINATORICS & PROBABILITY

COMBINATORICS

- [ ] **Permutations & Combinations:** Ordered arrangements $P(n,r)$ vs unordered selections $C(n,r) = n! / (r!(n-r)!)$.
- [ ] **Principle of Inclusion-Exclusion (PIE):** Correcting double/triple counts across 2, 3, or n overlapping sets.
- [ ] **Stars and Bars:** Distributing n identical items into k distinct bins: $C(n+k-1, k-1)$.
- [ ] **Combinatorial Identities:** Vandermonde's Identity, Hockey-Stick Identity, symmetry $C(n,k) = C(n, n-k)$.

- [ ] **Pascal's Triangle & Binomial Theorem:** Expanding $(x+y)^n$ using binomial coefficients $C(n,k)$.
- [ ] **Geometric Counting:** Grid path counting, counting sub-triangles/rectangles in geometric figures.
- [ ] **Recursion & Recurrence Relations:** Setting up homogeneous linear relations for state processes.

PROBABILITY & STATISTICS

- [ ] **Probability Rules & Conditional Probability:** Independent events, $P(A|B)$, and Bayes' Theorem.
- [ ] **Geometric Probability:** Length, area, and coordinate region volume probability calculations.
- [ ] **Expected Value:** $E[X] = \sum x_i \cdot P(x_i)$ and Linearity of Expectation $E[X+Y] = E[X] + E[Y]$.
- [ ] **Probability States & Markov Chains:** Solving infinite looping probabilities via linear system equations.
- [ ] **Mean, Median, Mode & Statistics:** Measures of central tendency, range, and weighted averages.

3. ALGEBRA & ADVANCED FUNCTIONS

ALGEBRAIC OPERATIONS & SEQUENCES

- [ ] **Algebraic Manipulations & Factorization:** Sum/difference of cubes ($a^3 \pm b^3$), symmetric expressions, completing square.
- [ ] **Systems of Equations:** Multi-variable linear & non-linear systems via substitution, elimination, matrices.
- [ ] **Speed, Distance, Time:** $d = rt$, relative motion, average speed, and river/current problems.
- [ ] **Arithmetic Sequences & Series:** $a_n = a_1 + (n-1)d$ and finite sum $S_n = (n/2)(a_1 + a_n)$.
- [ ] **Geometric Sequences & Series:** $a_n = a_1\, r^{n-1}$; sum $S_n = a(1-r^n)/(1-r)$; infinite sum $S_\infty = a / (1-r)$.
- [ ] **Special & Telescoping Sequences:** Fibonacci, Lucas numbers, telescoping series, arithmetico-geometric series.
- [ ] **Floor and Ceiling Functions:** Manipulating $\lfloor x \rfloor$ and $\lceil x \rceil$ in algebraic expressions and series.
- [ ] **Logarithms:** Log rules, change of base formula $\log_b(a) = \log_c(a)/\log_c(b)$, log equations.
- [ ] **Complex Numbers:** Operations in $a + bi$, magnitude $|z|$, Euler's formula $e^{i\theta}$, roots of unity.
- [ ] **Trigonometry (Algebraic & Geometric):** Double-angle, sum-to-product, Law of Sines ($a/\sin A = 2R$), Law of Cosines.

POLYNOMIALS & INEQUALITIES

- [ ] **Vieta's Formulas for Polynomials:** Relating coefficients to root sums and products for quadratics, cubics, n-degree.
- [ ] **Polynomial Remainder & Factor Theorem:** $P(a)$ is remainder when $P(x)$ divided by $(x-a)$; factor if $P(a) = 0$.
- [ ] **Simon's Favorite Factoring Trick (SFFT):** Factoring $xy + ax + by + ab = (x + b)(y + a)$ to solve integer equations.
- [ ] **AM-GM Inequality:** For positive real numbers: $(a_1 + a_2 + \ldots + a_n)/n \geq \sqrt[n]{(a_1\, a_2 \ldots a_n)}$.
- [ ] **Cauchy-Schwarz Inequality:** $(a_1^2 + a_2^2 + \ldots)(b_1^2 + b_2^2 + \ldots) \geq (a_1b_1 + a_2b_2 + \ldots)^2$.

4. NUMBER THEORY

CORE NUMBER THEORY

- [ ] **Primes & Factorization:** Prime factorization tree, total divisor count $(e_1+1)(e_2+1)\ldots$, sum of divisors.
- [ ] **Divisibility Rules:** Rules for 2, 3, 4, 5, 6, 7, 8, 9, 11, 13 and modular reduction properties.
- [ ] **GCD, LCM & Euclidean Algorithm:** Finding greatest common divisor via repeated division algorithm.
- [ ] **Modular Arithmetic Basics:** Remainder arithmetic, modular inverses, finding units digits / last digits.

- [ ] **Number Bases:** Converting integers/decimals between base-10 and base-b; alternate base operations.
- [ ] **Diophantine Equations:** Solving linear and quadratic equations restricted to integer solutions.
- [ ] **Algebraic Number Theory:** Gaussian integers $a + bi$ with integer components, norm properties.

NUMBER THEORY THEOREMS

- [ ] **Legendre's Formula:** Finding prime exponent in factorial: $E_p(n!) = \sum \lfloor n / p^k \rfloor$.
- [ ] **LCM-GCD Identity:** Product identity for two integers: $gcd(a,b) \cdot lcm(a,b) = a \cdot b$.
- [ ] **Fermat's Little Theorem:** If p is prime and $gcd(a,p) = 1$, then $a^{p-1} \equiv 1 \pmod p$.
- [ ] **Euler's Totient Theorem:** Generalization: $a^{\phi(n)} \equiv 1 \pmod n$ for coprime a and n.
- [ ] **Chicken McNugget Theorem:** Largest unobtainable integer for coprime a, b is $ab - a - b$.
- [ ] **Chinese Remainder Theorem (CRT):** Solving simultaneous system of linear congruences with coprime moduli.